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ABSORPTION OF PARTICLES IN FILTERS OF WATER-PIPELINE PURIFICATION WORKS

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A quantitative evaluation of the factors that can influence the absorption of particles is performed. It is shown that the approach of a particle to the surface of a grain of a charge is governed by the sedimentation and the position in the flow. The absorption itself is attributable to a sieve effect.

Introduction. Despite the fact that the theory of purification of water on filters has a long history, the situation regarding problems concerning the mechanism of absorption of particles is not entirely clear (see, e.g., [1-3]). In our opinion, a detailed analysis of the mechanism was not dictated by the inner logic of the development of the theory, since a decisive role was played by phenomenology and technological modeling.

At the present stage the situation is beginning to change. More stringent requirements on the quality of water and the sanitary hygienic reliability of the water supply induce a shift in interest toward methods of simulation modeling and computational experiments in the course of which one must keep track of the fate of each particle of impurity. And it is difficult to count on success here without a quantitative analysis of the contribution of possible factors to the absorption of particles. We made an attempt at such an evaluation.

Let us consider separately the phases of approach of a particle to the surface of a grain of the filter medium, absorption of particles, and formation of a precipitate in the pores.

1. Approach of a Particle to the Surface of a Grain. This process can be influenced by inertial, gravitational, diffusional, and molecular forces, the location in the flow, velocity fluctuations, rotation in the flow, and descent from a streamline.

1.1. The essential idea of inertial approach is that a particle does not move along a streamline, but descends from this line when it bends. For a particle of spherical shape the maximum inertial range is determined by the expression [4]

$$vd^2\Delta\rho/18\mu\,.$$

Having divided it by the characteristic dimension (which in the present case is equal to the grain diameter of the filter medium D), we obtain the Stokes number, which characterizes the effect of inertia on the motion of the particle:

$$\mathrm{St} = vd^2\Delta\rho/18\mu D\,. \qquad (1)$$

The velocity is assumed to be equal to the mean rate of filtrate flow, and the typical value $D = 1$ mm is taken. The value of $\Delta\rho$ does not exceed 180 kg/m^3 [5]. Then, for the maximum rate of filtrate flow (10 m/h) and a diameter of a particle of 100 μm

$$\mathrm{St} = 0.0027\cdot10^{-8}\cdot180/(18\cdot10^{-3}\cdot10^{-3}) = 0.027\cdot10^{-4}\,,$$

which is much smaller than the critical values of the Stokes number [6] at which the inertial forces begin to exert an influence.

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1.2. Sedimentation, i.e., gravity-induced descent of particles from the streamlines that envelope the grains, may be exhibited in a descending stream of water. We evaluate the influence of this factor in the following way.

We find the critical length L_{cr} over which sedimentation occurs. The latter is important if L_{cr} is equal to D in order of magnitude and is not important in the opposite case.

The stationary rate of sedimentation [4] is

$$v_{\mathrm{s}} = d^2 g\Delta\rho/18\mu . \tag{2}$$

Considering a pore channel as a tube of radius R, we will write an expression for the critical length [4]:

$$L_{\mathrm{cr}} = 8Rv/3v_{\mathrm{s}} . \tag{3}$$

Taking into account that $R \approx D/2$, for the minimum value of the velocity (5 m/h) and a diameter of a particle of 100 μm we obtain

$$L_{\mathrm{cr}} = 24Dv\mu/(d^2 g\Delta\rho) = 24\cdot 10^{-3}\cdot 0.00135\cdot 10^{-3}/(10^{-8}\cdot 9.8\cdot 180) \approx 2\cdot 10^{-3}\ \mathrm{m} .$$

The probability of approach (the efficiency of the sedimentation [7]) is

$$\eta_{\mathrm{sed}} = v_{\mathrm{s}}/v = 10^{-8}\cdot 9.8\cdot 180/(18\cdot 10^{-3}\cdot 0.00135) = 0.73 .$$

But already for a diameter of a particle of 30 μm the value of η_{sed} is an order of magnitude smaller. Thus, the contribution of sedimentation is important for large particles and unimportant for fine ones.

1.3. To evaluate the importance of diffusional transport of particles to the surface of a grain, we shall first evaluate the magnitude of the Schmidt number:

$$\mathrm{Sc} = \mu/\rho\mathscr{D}. \tag{4}$$

We find the coefficient of diffusion from the relation [4]

$$\mathscr{D}/kT = v_{\mathrm{s}}/mg . \tag{5}$$

For a spherical particle of diameter 1 μm and density 1180 kg/m^3

$$\mathscr{D} = kTv_{\mathrm{s}}/mg = 1.38\cdot 10^{-23}\cdot 293\cdot 10^{-12}\cdot 180\cdot 6/(18\cdot 10^{-3}\cdot \pi\cdot 10^{-18}\cdot 1180) \approx 6.5\cdot 10^{-14}\ \mathrm{m}^2/\mathrm{sec}$$

$$\mathrm{Sc} = 10^{-6}/6.5\cdot 10^{-14} = 0.15\cdot 10^{-8} .$$

At small values of Re and large Sc the number of particles settling on a sphere (a grain) in 1 sec is given by the Levich expression [8]:

$$\Phi \approx 8n_0\mathscr{D}^{2/3} v^{1/3} (D/2)^{4/3} . \tag{6}$$

The coefficient of n_0 is equal to $5.7\cdot 10^{-14}$, whence it is possible to conclude that diffusional transport is not important. Equation (6) is valid only for point particles [9]; nevertheless the "safety margin" seems to be sufficient.

1.4. Suppose a grain and a particle are separated by a water interlayer of thickness $h > 0.1$ μm, i.e., the electromagnetic lag is manifested completely. In view of the fact that $D >> d$, we represent the grain and the particle by a plane and a sphere, respectively. Then, the molecular attractive force per unit surface is

$$F = \pi dK/3h^3 .$$

With account for the force of hydrodynamic repulsion, the efficiency of the interaction has the form [10]

$$\eta_{\text{mol}} = 1.5\,(8\pi K/(9\mu v d^2 D))^{2/3}\,. \tag{7}$$

When the grain and the particle consist of quartz, $K \approx 10^{-28}$ J·m and for $d = 10\,\mu$m

$$\eta_{\text{mol}} = 1.5\,(8\pi\cdot 10^{-28}/(18\cdot 10^{-3}\cdot 0.0027\cdot 10^{-10}\cdot 10^{-3}))^{2/3} = 1.5\cdot 10^{-6}\,,$$

If the particles are flakes of coagulated suspension, the evaluation can be made differently. At the distances indicated the functional dependence of the force of molecular attraction on the static dielectric permittivity of the media involves the relation [11]

$$(\varepsilon_{10} - \varepsilon_{30})\,(\varepsilon_{20} - \varepsilon_{30})/((\varepsilon_{10} + \varepsilon_{30})\,(\varepsilon_{20} + \varepsilon_{30}))\,.$$

The flakes consist of more than 95% of water. The static dielectric permittivity of such systems virtually does not differ from that for bulk water, i.e., $\varepsilon_{20} = \varepsilon_{30}$, whence it follows that the force of molecular attraction of the flakes to the charge is equal to zero.

Thus, molecular attraction does not exert any influence on the approach of particles to a surface.

1.5. If the streamline along which a particle moves is separated from the surface of a grain by not more than some distance h_0, the particle touches the surface and comes into mechanical interaction with the latter. The fraction of such particles (the probability of contact) is equal to the ratio of the area of a zone of width h_0 and the cross-sectional area of a pore channel:

$$\eta_{\text{con}} = s/S\,, \tag{8}$$

where $s = \pi h_0^2 (2R/h_0 - 1)$; $S = \pi R^2$.

In order of magnitude $h_0 = d/2$, so that for particles of diameter 1–100 μm

$$\eta_{\text{con}} = (d/D)^2\,(2D/d - 1) = 2\,(10^{-1}\,...\,10^{-3})\,.$$

1.6. A particle can find itself near the surface of a grain as a result of flow rate fluctuation due to the irregularity of the pore space. The characteristic time of fluctuation is of the order of D/v, i.e., 0.37–0.74 sec.

On the other hand, it is known [12] that the characteristic time needed by a particle that participates in unsteady motion to attain a stationary velocity is $(d/2)^2\rho/\mu$ in order of magnitude, i.e., 10^{-7}–10^{-3} sec.

From this it follows that velocity fluctuations do not influence the absorption of particles.

1.7. The factors considered above are virtually insensitive to small departures of the shape of the particles from spherical. In fact, the shape factor of natural particles is $f \approx 1.2$–1.5, and for flakes it is $f \approx 2$; therefore they do not rotate in a flow of liquid, and platy (flaky) particles are oriented parallel to the streamlines [4].

The evaluatons carried out show that the approach of a particle to the surface of a grain is caused only by sedimentation and the location in the flow. By analogy with [7], the probability of approach η has the form

$$\eta = 1 - (1 - \eta_{\text{sed}})\,(1 - \eta_{\text{con}})\,. \tag{9}$$

2. Absorption of Particles. Approach to a surface does not mean absorption as yet. Absorption proper may occur due to molecular forces, a frictional force, and a sieve effect.

2.1. Molecular forces of attraction and forces of repulsion are always present between two bodies. The resulting specific energy of interaction is characterized by a secondary (long-range) minimum at a certain distance between the bodies [11]. If the bodies come within this distance of each other, we may speak of their mutual fixation and capture of one body by the other.

For a quartz-water-quartz system it is shown that the long-range minimum of the energy of interaction occurs at $h = 50$–175 Å [13-15]. In order of magnitude this is much smaller than the characteristic dimensions of the roughnesses (microinhomogeneities) of the surface of both a grain and a particle (0.1–10 μm). Consequently, the roughness of the surface prevents a particle from coming within the distance at which it could be captured by the surface of a grain due to molecular forces.

When the particles consist of flakes, the arguments given in Subsec. 1.4 concerning the equality of molecular forces to zero remain valid.

2.2. The interaction of the roughnesses of the surfaces leads to the appearance of a frictional force between a particle and a grain that is directed opposite to the flow velocity. The efficiency of absorption due to the frictional force can be conveniently evaluated from expression (8) in which s is now the dimensional cross section of absorption.

From the side of the flow a particle experiences the action of a Stokes force. In a cylindrical coordinate system it has the form

$$F = 3\pi d\mu v(r) \,,$$

where $v(r) = 3v(1 - r^2)/R^2)/2$; $h_0 = r - r_0$; here r_0 is determined from the equality of the Stokes and frictional forces in the spirit of [16]:

$$F_{\text{fr}} = 9\pi d\mu v(1 - r_0^2/R^2)/2 \,,$$

whence

$$h_0 = R(1 - (1 - 2F_{\text{fr}}/9\pi d\mu v)^{1/2} \,. \tag{10}$$

After substitutions, the efficiency of absorption due to the frictional force is

$$\eta_{\text{fr}} = 2F_{\text{fr}}/9\pi d\mu v \,. \tag{11}$$

To evaluate the frictional force, we avail ourselves of an equation from the theory of soils [17]:

$$\tau = \sigma \tan \varphi \,. \tag{12}$$

If it is assumed that a normal stress is generated by the hydrodynamic pressure of the liquid, which presses a particle against the surface of a grain, and it is taken that $\varphi = 30^0$, then at the characteristic (maximum) rate of filtrate flow we obtain $\tau < 4 \cdot 10^{-3}$ Pa, $F_{\text{fr}} \sim 10^{-17}$ N, and $\eta_{\text{fr}} \sim 1.5 \cdot (10^{-7} - 10^{-6})$ for particles of diameter 1–10 μm even without account for the increase in the Stokes and hydrodynamic repulsive forces caused by the presence of a solid boundary [18].

The evaluation given is valid for dust and silt particles. For flakes of a coagulated (flocculated) suspension it is simpler to find the shear stress experimentally by rheological methods; it may attain 1 Pa, and correspondingly $\eta_{\text{fr}} = 10^{-4} - 10^{-3}$.

Above, the applicability of the Reynolds approximation was implied everywhere for the force of resistance of a spherical particle, although in a filter Re $\approx$ 1 and formally one should take the Oseen approximation [8]

$$F = 3\pi d\mu v(1 + 3d\rho v/16\mu) \,. \tag{13}$$

For a 1-μm particle the error is 0.05% and for a 100-μm particle it is 5%, so that the application of the Reynolds approximation is entirely justified.

2.3. The unimportance of the frictional force is confirmed by experiments carried out by B. A. Mitin. According to these experiments, particles that approach the frontal surface of a grain do not remain on it, but "slide" to zones of contact of the grains. There they fall into gaps between grains or into mouths of pores, which can be considered to be effective centers of interaction of particles with the charge, i.e., traps, and the very process of motion of particles can be considered to be a wandering in the system of these traps [19].

Absorption begins from the jamming of a trap (sieve effect). The condition of jamming has the form $l_p \geq l$ for anisometric particles or a single particle and $\Sigma d^2 \geq l^2$ for several isometric particles. For subsequent particles entry into a jammed trap up to exhaustion of its capacity is equivalent to absorption with probability (9).

3. **Formation of a Precipitate.** The particles absorbed consolidate with time, and thus formation of a precipitate occurs. This physicochemical process can be considered as a continuation of the reaction (an increase in the degree) of polymerization of a coagulant/flocculant.

The reaction rate constant is affected by the chemical composition of water and the reagents, the pH, the nature and state of the surface of the charge grains, and the temperature. Under real conditions the influence of the majority of the factors is uncontrolled. Therefore it is difficult to give a rather accurate theoretical description of the dynamics of formation of a precipitate. But it is comparatively easy to do this experimentally by rheological methods [20].

By analogy with the transition from unbound to bound soil, formation of a precipitate means the appearance of a structure, i.e., a change in the strength. Accordingly, in expression (12) for the shear stress, which characterizes the strength, a second term, namely adhesion, appears:

$$\tau = \sigma \tan \varphi + c .$$

Measuring the static shear stress, it is possible to judge the state of the structure of the precipitate. In order of magnitude, the static shear stress of a fresh precipitate is equal to 1–2 Pa.

Equality of the shear stress and the shearing stress $\mu\Gamma$, where Γ is the velocity gradient in the flow for the given rate of filtering, determines the equilibrium boundary between the absorbed particles and the flow, i.e., the capacity of the traps.

Conclusions. The quantitative evaluation of the factors that can influence the absorption of particles in filters of water-pipeline purification works showed that the approach of a particle to the surface of a charge grain is attributable to sedimentation and the location in the flow. Absorption proper is associated with the sieve effect. The number of particles absorbed is determined, on the one hand, by the shear stress and, on the other hand, by the rate of filtering flow.

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NOTATION

v, speed of motion; d, diameter of a particle; $\Delta\rho$, difference between the densities of a particle and water; μ, dynamic viscosity of water; D, diameter of a grain of the filter medium; L_{cr}, critical length; v_s, stationary rate of sedimentation; g, acceleration of gravity; ε, density of water; $\mathcal{D}$, coefficient of diffusion; k, Boltzmann's constant; T, absolute temperature; m, mass of a particle; n_0, concentration of particles in he flow; ε_{10}, ε_{20}, ε_{30}, static dielectric permittivity of the material of a grain, a particle, and bulk water; τ, shear stress; σ, normal stress; φ, angle of repose; l, l_p, characteristic dimension of a mouth and a particle; c, adhesion.

REFERENCES

1. E. V. Venitsianov and R. N. Rubinshtein, Dynamics of Sorption from Liquid Media [in Russian], Moscow (1983).
2. A. V. Sandulyak, Purification of Liquids in a Magnetic Field [in Russian], L'vov (1984).
3. I. M. Leichkis and I. M. Fedotkin, Ukr. Khim. Zh., 43, No. 2, 40-43 (1977).
4. N. A. Fuks, Mechanics of Aerosols [in Russian], Moscow (1955).
5. E. D. Babenkov, Purification of Water by Coagulants [in Russian], Moscow (1977).
6. L. M. Levin, Investigations of the Physics of Coarse-Disperse Aerosols [in Russian], Moscow (1961).
7. V. N. Uzhov and B. I. Myagkov, Cleaning of Industrial Gases by Filters [in Russian], Moscow (1970).
8. N. A. Fuks, Progress in the Mechanics of Aerosols [in Russian], Moscow (1961).
9. G. L. Natanson, Dokl. Akad. Nauk SSSR, 112, No. 1, 100-103 (1955).

10. B. V. Deryagin and L. P. Smirnov, in: Investigations in the Field of Surface Forces [in Russian], Moscow (1967), pp. 188-206.
11. B. V. Deryagin, N. V. Churaev, and V. M. Muller, Surface Forces [in Russian], Moscow (1985).
12. B. I. Brounshtein and G. A. Fishbein, Hydrodynamics and Mass and Heat Transfer in Disperse Systems [in Russian], Leningrad (1977).
13. Ya. I. Rabinovich and N. N. Churaev, Kolloid. Zh., **11**, No. 3, 168-173 (1979).
14. Ya. I. Rabinovich, Kolloid. Zh., 39, No. 2, 1094-1099 (1977).
15. N. V. Churaev, in: Physicochemical Mechanics and the Lyophilic Nature of Disperse Systems [in Russian], Kiev (1989), Issue 20, pp. 10-19.
16. Ts. E. Mirtskhulava, Fundamentals of the Physics and Mechanics of the Erosion of Beds [in Russian], Leningrad (1988).
17. M. P. Lysenko, Composition and Physicomechanical Properties of Soils [in Russian], Moscow (1980).
18. S. L. Goren and M. E. O'Neill, Chem. Eng. Sci., 26, 325-338 (1971).
19. G. D. Reznikov, Mat. Modelir., 7, No. 5, 119-127 (1995).
20. G. D. Reznikov, Inzh.-Fiz. Zh., 55, No. 1, 147 (1988).